In this recording, I'm going to show how to calculate eigenvectors for a 3 by 3 matrix. I'm going to use the same matrix whose eigenvalues are already found in the previous recording. The matrix was the A, as written here. Recall that the eigenvalues we found were 1, 2, and negative 1.

I've also written down the combination A minus lambda I in terms of lambda. You're going to have to take lambda equal to each of eigenvalues in turn, and look at three different cases to find the three different eigenvectors.

Recall that the eigenvectors-- I'll call them v-- satisfy the equation that, when multiplied on the left by the matrix A minus lambda I, they must give 0. I write my v as having components v1, v2, v3, and consider it to be a column.

The equations we have to solve will be equations for v1, v2, v3-- linear simultaneous equations. We can solve those by Gaussian elimination.

Let's write down the basic matrix for which we're going to have to do the elimination. It will look like this. We'll have to do row operations to try and find values for v1, v2, and v3. And we'll have to do them three different times for each of the different lambdas.

Let's start with lambda equals 1. We'll call it Case One. I'll just have to put lambda equals 1 in the matrix above. That will give me 0, 1, negative 2, negative 1, 1, 1, and a 0, 1, negative 2. Put the column of 0s on the right, of course.

The elimination process is not too hard for this matrix because we've already got some nice 0s. I'd like a 1 to be in the top left-hand corner. So I'm going to multiply my second row by negative 1, and then swap it with the first row. Here's what we get. Remember, I'm multiplying the second row by negative 1, but also changing the sign. That will be 1, negative 1, negative 1. Got some 0s still. And the rows have got swapped.

I suppose I probably ought to say what I've done. So let's see here. Row two becomes negative row 2. And then my notation for swapping is to use a squiggle like this and put the rows in. So R1 and R2 are swapped.

Now look. We've got two rows the same. The third row could be set 0. R3 becomes R3 minus R2. So that now gives us-- leaving the first row the same. So the second row. The third row completely
disappears. We're happy to see that row of 0s there. That means we can find a non-trivial eigenvector.

We could now solve the equations. But I think I'm going to do one more Gaussian step. Look at the middle column-- negative 1 and 1. We could get rid of the negative 1 on top by adding row 2. So let's do becomes-- row 1 becomes row 1 plus row 2. That makes the result 1, 0, negative 3. With 0s still down the right. And everything else remains the same.

Now I've got a nice form for solving my equations. What do the equations say? The top row says v1 minus 3v3 equals 0. And second row says v2 minus 2v3 equals 0. So we can express everything in terms of v3. v1 is 3v3, and v2 is 2v3.

Let's now write down our eigenvector. v must be v1 on top, v2 in the middle, v3 at the bottom. And we could pull the v3 out as a factor. And we've got 3, 2, 1.

And then, because it's only the direction we care about and not the length, we can choose v3. And the sensible choice here would be to choose v3 just to be 1. So my eigenvector for the eigenvalue lambda equals 1 is just 3, 2, 1.

So this is the first time we've done this. So let's check it. Let's go back and write down the matrix and multiply it by the v. Here's the multiplication we have to do. We hope we're going to get the answer 3, 2, 1 again, because the eigenvalue is 1. Let's do the matrix multiplication.

So my answer is a column vector formed from 3 plus 2 minus 2. Negative 3 plus 4 plus 1. And 2 minus 1. That simplifies to 3, 2, 1. Sure enough, we've got the eigenvalue 1 times our vector, 3, 2, 1.

Let's now repeat the process for the second eigenvalue. Second eigenvalue was lambda equals negative 1. We need to go back and substitute that value of lambda into our Gaussian elimination equation. That was back here. Perhaps I should have marked it as star. No, it's not an equation. It's a matrix.

OK. I've got to put lambda equals negative 1. That's going to give me 2, 1, negative 2 across the top. Let's write that down. There's only going to be 0s in the column to the right, so I might as well put those in. Now let's go back and look at the middle row. When lambda is negative 1 and star, we have negative 1, 3, 1. And finally, the bottom row. Still in star, with lambda equal to negative 1. We get 0, 1, 0.

We have to do Gaussian elimination on this one. But actually, there's not much to do. Because that bottom row is already rather friendly. It's telling us v2 equals 0. Now to put the other two rows. With v2 equal to 0, we can drop the v2 terms. So the top row tells us 2v1 minus 2v3 equals 0. And the middle row tells us negative v1 plus v3 equals 0.

Those two equations are actually the same. The first one is just the second one multiplied by negative 2. So actually, we just see that the answer is v1 equals v3 with, at the same time, v2 equals 0.

Let's put those together into an eigenvector. If v1 is v3 and v2 is 0, it'll look like this. Once again, we could pull out a factor of v3 and get 1, 0, 1. Then we could let v3 be a number of choice. I'm going to choose 1 again, for convenience. So the eigenvector for lambda equals negative 1 is just 1, 0, 1.

I'm going to leave you to do the check this time. You multiply the matrix A by 1, 0, 1, and you should get negative 1 times 1, 0, 1. It does work. But you try it.
I'm also going to leave it as an exercise for you to find the last eigenvector. I'll write down the matrix that you have to do the Gaussian elimination on, and I'll tell you the answer, but I'd like you to try and derive it yourself.

So here we are. The eigenvalue is 2. The Gaussian elimination needs to be done on the matrix I've written here. And then eigenvector we find is 1, 3, 1. But I'll leave that for you to check, like I said.